Note on Symmetric Equilibria with Habit Formation

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To see that a Pareto-optimal symmetric equilibrium cannot be sustained in the context with persistent preferences and no entry, suppose that there exists a wage w set by all firms, and that, as in Card, Cardoso, Heining and Kline (2018), the indirect utility for worker i of working at firm j in time t is given by

$$u_{ijt} = \beta \log(w_{jt} - b) + a_j + h_t \mathbb{1}(M_{i,t-1} = j)\epsilon_{ij}$$

Suppose first that $h_0 = 0$ and $w_0 = \bar{w}$ for all j. Then if $h_1 = h > 0$, the elasticity of labor supply to the firm is 0 for wage increases or decreases satisfying

$$\left| \beta \left(\log \left(\frac{\bar{w} - b}{w_1 - b} \right) \right) \right| \leqslant h$$

Therefore, a profit maximizing firm will prefer w_1 given by

$$\beta \log \left(\frac{\bar{w} - b}{w_1 - b} \right) = h$$

$$w_1 = (\bar{w} - b) \exp \left(-\frac{h}{\beta} \right) + b < \bar{w}$$

And if there's a symmetric equilibrium at w_1, then again the labor supply curve is perfectly inelastic and the firm would choose

$$w_1' = \left(w_1' - b \right) \exp \left(-\frac{h}{\beta} \right) + b < w_1'$$

Therefore, the only symmetric equilibrium that can be sustained is $w_1 = b$ and the labor supply to the market is 0.

Now, consider the case where $h_0 > 0$. We want to show that any symmetric equilibrium is either identical to the h_0 case (workers wort based on their ϵ_{ij} values to their best matches) or the equilibrium isn't Pareto-optimal. But this follows directly from the fact that, if $\epsilon_{ij} < \epsilon_{ik}$ for some j and k, it is a Pareto improvement for that worker to begin at firm k in time 0 because there are constant returns to scale.

The intuition here is that a symmetric equilibrium cannot be sustained, because if all firms are offering the same wages, and workers at firm j have a particular preference for staying there due to the habit stock in their preferences, then firm j has an incentive to deviate and capture that preference value in by paying lower wages. But if every firm lowers their wages like that, so that the equilibrium is again symmetric, then firm j should again deviate and lower their wages further. Therefore, if there is a symmetric equilibrium, it involves firms bidding workers' wages down to their non-market outside option b. But one firm can deviate from this and set wages, say, at the monopsony wage without habits, and attract a large share of the workers. So, as in Burdett and Mortensen (1998), some firms choose to be large and have low markdowns while other firms are small but have high profits per worker. Another way to see this is that the kink in the labor supply curve facing the firm creates non-convexities and therefore multiple local maxima in the profits function.